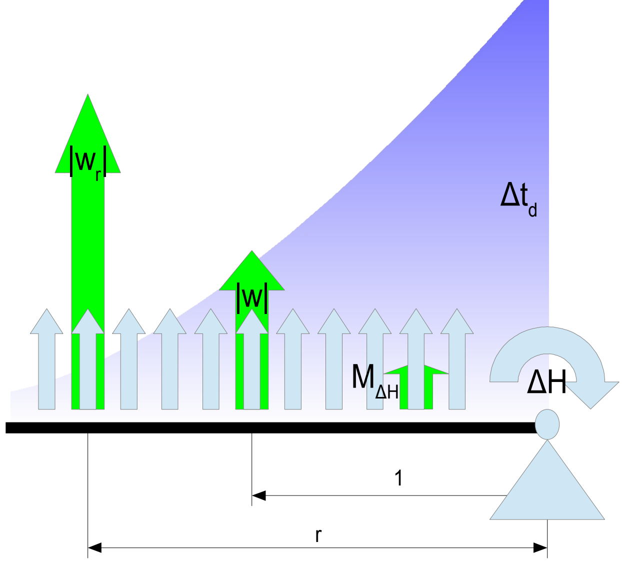
Dan Andersson, daaf11@student.bth.se 2014

Mathematical treatise: Fixed time-step issue

Angular collision response have a dynamic contact time depending on distance to the torque axis.



ΔH : Angular momentum acceleration.
$\vec{r}$: Offset to point of contact.
$M_{\Delta H}$: Particle momentum.
$\vec{w}$ & $\vec{w}_{\vec{r}}$: Angular velocity.
Δs : Collision contact stretch, relative to $M_{\Delta H}$.
Δt_f : Fixed time periodicy of simulation.
$\Delta t_d(\vec{r})$: Real contact duration time.
$c_{\Delta t} = \Delta t_d / \Delta t_f$: Regulating time coefficient.

Objective

Present a formula for the $c_{\Delta t}$ coefficient, that can be implemented into a 3D physics engine with fixed time periodicy.

Calculus

$$c_{\Delta t} = \frac{\Delta t_d(\vec{r})}{\Delta t_f}, \quad \Delta t_f \text{ is a constant.}$$

$$\vec{w}_{\vec{r}} = \left(H'_x \frac{(r'_y)^2 + (r'_z)^2}{I_{Sxx}}, \quad H'_y \frac{(r'_x)^2 + (r'_z)^2}{I_{Syy}}, \quad H'_z \frac{(r'_x)^2 + (r'_y)^2}{I_{Szz}} \right)$$ [1]

$$\vec{w} = \left(H'_x \frac{1}{I_{Sxx}}, \quad H'_y \frac{1}{I_{Syy}}, \quad H'_z \frac{1}{I_{Szz}} \right)$$

$$\begin{aligned} \Delta s &= \Delta t_d(\vec{r}) * |\Delta \vec{w}_{\vec{r}}| \\ \Delta s &= \Delta t_f * |\Delta \vec{w}| \end{aligned} \quad \rightarrow \quad \Delta t_d(\vec{r}) * |\Delta \vec{w}_{\vec{r}}| = \Delta t_f * |\Delta \vec{w}| \quad \rightarrow \quad \frac{\Delta t_d(\vec{r})}{\Delta t_f} = \frac{|\Delta \vec{w}|}{|\Delta \vec{w}_{\vec{r}}|} = c_{\Delta t}$$

$$c_{\Delta t} = \frac{|\Delta \vec{w}|}{|\Delta \vec{w}_{\vec{r}}|}$$

$\vec{r}$, $\Delta \vec{w}$ and $\Delta \vec{H}$ is known and $\Delta \vec{w}_{\vec{r}}$ can be derived from those three.

Pseudo Code

if (|vecR| > 1) vecDeltaH * = AngularImpactTimeCoefficient(vecR, vecDeltaW, vecDeltaH);

Optimization

$c_{\Delta t} = |\Delta \vec{w}| / |\Delta \vec{w}_{\vec{r}}|$ can be a little bit encumbersome for a game engine. An approximative possibly good-enough alternative could be $c_{\Delta t} \sim 1/|\vec{r}|^2$.

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1 Converting angular momentum [kg*m^2/s] to angular velocity [radians/s] by Dan Andersson