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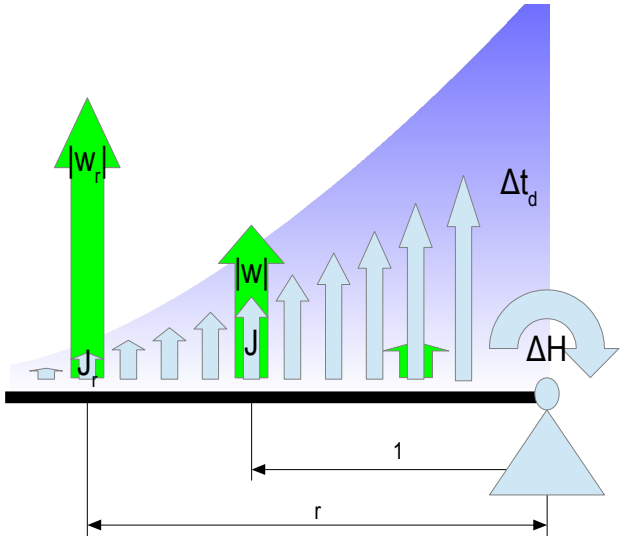
Mathematical treatise: Fixed time-step issue

Angular collision response have a dynamic contact time depending on distance to the torque axis.



ΔH : Change in angular momentum.
$\vec{r}$: Offset to point of contact.
$J_{\vec{r}}$: Particle momentum.
$\vec{w}$ & $\vec{w}_{\vec{r}}$: Angular velocity.
Δs : Collision contact stretch, relative to $M_{\Delta H}$.
Δt_f : Fixed time periodicy of simulation.
$\Delta t_d(\vec{r})$: Real contact duration time.
$c_{\Delta t} = \Delta t_d / \Delta t_f$: Regulating time coefficient.

Objective

Present a formula for the $c_{\Delta t}$ coefficient, that can be implemented into a 3D physics engine with fixed time periodicy.

Calculus

$$c_{\Delta t} = \frac{\Delta t_d(\vec{r})}{\Delta t_f}, \quad \Delta t_f \text{ is a constant.}$$

$$|\vec{w}_{\vec{r}}| = \left| \left(H'_x \frac{(r'_y)^2+(r'_z)^2}{I_{Sxx}}, \quad H'_y \frac{(r'_x)^2+(r'_z)^2}{I_{Syy}}, \quad H'_z \frac{(r'_x)^2+(r'_y)^2}{I_{Szz}} \right) \right| \quad [1]$$

$$|\vec{w}| = \left| \left(H'_x \frac{6}{I_{Sxx}}, \quad H'_y \frac{6}{I_{Syy}}, \quad H'_z \frac{6}{I_{Szz}} \right) \right|$$

$$\left\{ \begin{array}{l} \Delta s = \Delta t_d(\vec{r}) * |\Delta \vec{w}_{\vec{r}}| \\ \Delta s = \Delta t_f * |\Delta \vec{w}| \end{array} \right\} \rightarrow \Delta t_d(\vec{r}) * |\Delta \vec{w}_{\vec{r}}| = \Delta t_f * |\Delta \vec{w}| \rightarrow \frac{\Delta t_d(\vec{r})}{\Delta t_f} = \frac{|\Delta \vec{w}|}{|\Delta \vec{w}_{\vec{r}}|} = c_{\Delta t}$$

$$c_{\Delta t} = \frac{|\Delta \vec{w}|}{|\Delta \vec{w}_{\vec{r}}|} \quad \vec{r}, \Delta \vec{w} \text{ and } \Delta \vec{H} \text{ is known and } \Delta \vec{w}_{\vec{r}} \text{ can be derived from those three.}$$

Pseudo Code

```
if ( |vecR| > 1 )  vecDeltaH * = AngularImpactTimeCoefficient(  vecR, vecDeltaW, vecDeltaH );
```

Optimization

$c_{\Delta t} = |\Delta \vec{w}| / |\Delta \vec{w}_{\vec{r}}|$ can be a little bit encumbersome for a game engine. An approximative possibly good-enough alternative could be $c_{\Delta t} \sim 1/|\vec{r}|^2$.

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1 Converting angular momentum [kg*m^2/s] to angular velocity [radians/s] by Dan Andersson